P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
II year B.Sc., Degree Examinations - III Semester
Mathematics Course-III: ABSTRACT ALGEBRA
(w.e.f. 2020-21 Admitted Batch)

Total Hrs. of Teaching-Learning: 75 @ 6 hr/Week **Total credits: 04**

Objective:

- To learn about the basic structure in Algebra
- To understand the concepts and able to write the proofs to theorems
- To know about the applications of group theory in real world problems

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Course Outcomes:

After successful completion of this course, the student will be able to;

- acquire the basic knowledge and structure of groups, subgroups and cyclic groups.
- get the significance of the notation of a normal subgroups.
- get the behavior of permutations and operations on them.
- study the homomorphisms and isomorphisms with applications.
- Understand the ring theory concepts with the help of knowledge in group theory and to prove thetheorems.
- Understand the applications of ring theory in various fields.

UNIT I : **(12 Hours)**

GROUPS : Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

UNIT II: **(12 Hours)**

SUBGROUP:Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition- examples-criterion for a complex to be a subgroups. Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. Co-sets and Lagrange's Theorem: Cosets Definition

– properties of Cosets–Index of a subgroups of a finite groups–Lagrange's Theorem.

UNIT III: **(12 Hours)**

NORMAL SUBGROUPS: Definition of normal subgroup – proper and improper normal subgroup– Hamilton group – criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups – Sub group of index 2 is a normal sub group –quotient group – criteria for the existence of a quotient group

UNIT IV: **(12 Hours)**

HOMOMORPHISM : Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

PERMUTATIONS: Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

UNIT V: (12 Hours)

RINGS

Definition of Ring and basic properties, Boolean Rings, divisors of zero and cancellation laws Rings,Integral Domains, Division Ring and Fields, The characteristic of a ring - The characteristic of an Integral Domain, The characteristic of a Field. Sub Rings.

Co-Curricular Activities(15 Hours)

Seminar/ Quiz/ Assignments/ Group theory and its applications / Problem Solving.

TEXT BOOK :

1. A text book of Mathematics for B.A. / B.Sc. by B.V.S.S. SARMA and others, published byS.Chand & Company, New Delhi.

REFERENCE BOOKS :

1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.
2. Modern Algebra by M.L. Khanna.
3. Rings and Linear Algebra by Pundir & Pundir, published by Pragathi Prakashan.

Additional Inputs ;

Cyclic groups definition and number of generators of a finite cyclic groups .

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-III

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Groups	2	2	30
II	Subgroups	2	2	30
III	Normal subgroups	1	2	25
IV	Homomorphism, Permutations	2	2	30
V	Rings	1	2	25
Total		8	10	140

S.A.Q. **=** Short answer questions (5 marks)

E.Q **=** Essay questions (10 marks)

Short answer questions : 4 X 5 = 20

Essay questions : 4 X 10 = 40

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Total Marks = 60

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P.R. Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Mathematics Course: Abstract Algebra
Paper III (Model Paper w.e.f. 2020-21)

Time: 2Hrs 30 min **Max. Marks: 60**

PART - I

Answer any FOUR questions. Each question carries FIVE marks. **4 X 5 M=20 M**

1. Prove that the set Z of all integers form an abelian group w.r.t. the operation defined by $a * b = a + b + 2, \forall\, a, b \in Z$
2. If G is a group,for $a, b \in G$ prove that $(ab)^{-1} = b^{-1}a^{-1}$
3. If a non empty complex H of a group G is a subgroup of G then prove that $H = H^{-1}$.
4. Prove that a non empty finite complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab \in H$.
5. Define Normal subgroup. Prove that a subgroup H of a Group (G,.) is a normal subgroup of G if and only if $xHx^{-1} = \mathrm{H}\ \forall\ \mathrm{x} \in \mathrm{G}$.
6. If f is a homomorphism of a group G into a group G',then prove that the kernel of f is a normal subgroup of G.
7. Write down the following permutation as product of disjoint cycles
$$f = (1\ 3\ 2\ 5)(1\ \ 4\ \ 3)(2\ \ 5\ \ 1).$$
8. Show that a ring R has no zero divisors if and only if the cancellation laws hold in R.

PART - II

Answer ALL questions. Each question carries Eight marks. **5 X 8 M = 40 M**

9 **(a)** Prove that the set Z of all integers form an abelian group w.r.t. the operation is defined by $a * b = a + b + 2\ \forall\, a, b \in Z$.

(OR)

(b) Show that the set Q_+ of all positive rational numbers forms an abelian group under the composition defined by 'o' such that $a\, ob = (ab)/3\ \ for\ a, b\ \in\ Q_+$.

10 (a)Prove that a non empty complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow\ \ ab^{-1} \in H$.

(OR)

(b) State and prove Lagrange's Theorem. Prove that the converse of Lagrange's theorem is not true

11 **(a)** If H is a normal subgroup of a group (G,.) then prove that the product of two right (or) left cosets of H is also a right (or) left coset of H in G.

(OR)

(b) If H is a subgroup of G and N is a normal subgroup of G, then prove that
(i) $H \cap N$ is a normal subgroup of H (ii) N is a normal subgroup of HN

12 .(a) Prove that the necessary and sufficient condition for a homomorphism f of a group G onto a group G' with kernel K to be an isomorphism of G into G' is that $K = \{e\}$

(OR)

(b) $f = (1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8)$, $g = (4\ \ 1\ \ 5\ \ 6\ \ 7\ \ 3\ \ 2\ \ 8)$ are cyclic permutations. Show that $(fg)^{-1} = g^{-1}f^{-1}$.

13 .(a) A finite integral domain is a field .

(OR)

(b) Prove that the ring of integers Z is a principal ideal ring.